Fuzzy Signature and Cognitive Modelling for Complex Decision Model

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Abstract. As data is getting more complex and complicated, it is increasingly difficult to construct an effective complex decision model. Two very obvious examples where such a need emerges are in the economic and the medical fields. This paper presents the fuzzy signature and cognitive modeling approach which could improve such decision models. Fuzzy signatures are introduced to handle complex structured data and problems with interdependent features. A fuzzy signature can also be used in cases where data is missing. The proposed fuzzy signature structure will be used in problems that fall into this category. This paper also investigates a novel cognitive model to extend the usage of fuzzy signatures. This Fuzzy Cognitive Signature Modelling will enhance the usability of fuzzy theory in modelling complex systems as well as facilitating complex decision making process based on ill structured information or data.

1 Introduction

Fuzzy control and decision support systems are still the most important applications of fuzzy theory [1, 2]. This is a general form of expert control using fuzzy sets representing vague / linguistic predicates, modelling a system by If ... then rules. In the classical approaches of Zadeh [3] and Mamdani [4], the essential idea is that an observation will match partially with one or several rules in the model, and the conclusion is calculated by evaluation of the degree of these matches and by the use of the matched rules.

Fuzzy modelling has become popular because of its ability to assign meaningful linguistic labels to the fuzzy sets [5] in the rule base [6, 7]. However, a serious problem is caused by the high computational time and space complexity of rule bases describing systems with multiple inputs with proper accuracy. The complexity allows little general systems application (or real time control application) of classical fuzzy algorithms, where the inputs exceed about 6 to 10. These traditional fuzzy systems deal with very simple structured data, where the number of inputs is well defined, and values for each input occur for most or all data items. This further reduces their general applicability.

O. Castillo et al. (Eds.): Theor. Adv. and Appl. of Fuzzy Logic, ASC 42, pp. 380–389, 2007.
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Basically, practical fuzzy rule bases suffer from rule explosion. The number of possible rules necessary is $O(T^k)$ where K is the number of dimensions and T is the number of terms per input. In order to increase the problems solvable by fuzzy rule-base systems, it is essential to reduce T, k, or both. Decreasing T leads to sparse fuzzy systems, i.e. fuzzy rule-bases with "gaps" between the rules [8]. On the other hand, decreasing K leads to hierarchical fuzzy systems [9, 10].

A signature, as an abbreviated but unambiguously characteristic reference to data is widely used in computer based applications for data organization, retrieval, data mining. The abbreviation and conceptual clustering nature also suggests the use of fuzzy signatures. Fuzzy signatures create a natural bridge to verbal classifications, and human estimations. Fuzzy signatures which structure data into vectors of fuzzy values, each of which can be a further vector, are introduced to handle complex structured data [11, 12]. This will widen the application of fuzzy theory to many areas where objects are complex and sometimes interdependent features are to be classified and similarities / dissimilarities evaluated. Often, human experts can and must make decisions based on comparisons of cases with different numbers of data components, with even some components missing. This fuzzy signature tree structure is a generalisation of fuzzy sets and vector valued fuzzy sets in a way modelling the human approach to complex problems.

When dealing with a very large data set, it is possible that there is hidden hierarchical structure that appears in the sub-variable structures. This paper is used to address problems having this characteristic, and the possible use of cognitive modeling in describing the relationships of multiple fuzzy signatures.

2 Fuzzy Signature

The original definition of fuzzy sets was $A : X \to [0,1]$, and was soon extended to *L-fuzzy sets* by Goguen [13],

$$A_S : X \to [a_i]_{i=1}^{k}, a_i = \begin{cases} [0,1] \\ [a_{ij}]_{j=1}^{k_i} \end{cases}, a_{ij} = \begin{cases} [0,1] \\ [a_{ijl}]_{l=1}^{k_{ij}} \end{cases},$$

$A_L : X \to L$, L being an arbitrary algebraic lattice. A practical special case, *Vector Valued Fuzzy Sets* was introduced by Kóczy [14], where $A_{V,k} : X \to [0,1]^k$, and the range of membership values was the lattice of k-dimensional vectors with components in the unit interval. A further generalisation of this concept is the introduction of fuzzy signatures and signature sets, where each vector component is possibly another nested vector (right).

Fuzzy signature can be considered as special multi-dimensional fuzzy data. Some of the dimensions are inter-related in the sense that they form sub-group of variables, which jointly determine some feature on a higher level. Let us consider an example. Figure 1 shows a fuzzy signature structure.

The fuzzy signature structure shown in Figure 1 can be represented in vector form as follow:

$$x = \begin{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \\ \begin{bmatrix} x_{21} \\ \begin{bmatrix} x_{221} \\ x_{222} \\ x_{223} \end{bmatrix} \\ x_{23} \end{bmatrix} \\ \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} \end{bmatrix}^T$$

Here $[x_{11}\ x_{12}]$ form a sub-group that corresponds to a higher level compound variable of x_1. $[x_{221}\ x_{222}\ x_{223}]$ will then combine together to form x_{22} and $[x_{21}[x_{221}\ x_{222}\ x_{223}]x_{23}]$ is equivalent on higher level with $[x_{21}\ x_{22}\ x_{23}] = x_2$. Finally, the fuzzy signature structure will become $x = [x_1\ x_2\ x_3]$ in the example.

The relationship between higher and lower level is govern by a set of fuzzy aggregations. The results of the parent signature at each level are computed from their branches with appropriate aggregation of their child signature. Let a_1 be the aggregation associating x_{11} and x_{12} used to derive x_1, thus $x_1 = x_{11}\, a_1\, x_{12}$. By referring to Figure 1, the aggregations for the whole signature structure would be a_1, a_2, a_{22}, and a_3. The aggregations a_1, a_2, a_{22}, and a_3 are not necessarily identical or

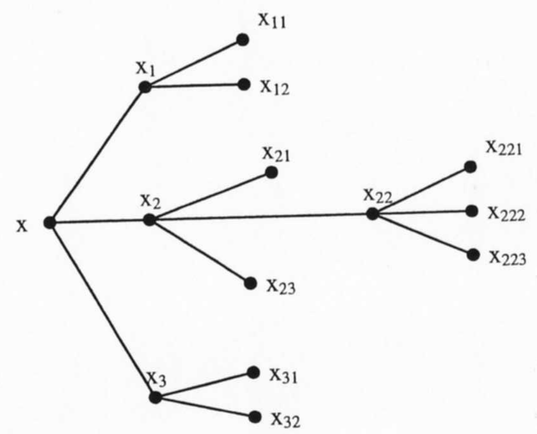

Fig. 1. A Fuzzy Signature Structure

different. The simplest case for a_{22} might be the *min* operation, the most well known t-norm. Let all aggregations be *min* except a_{22} be the averaging aggregation. We will show the operation based on the following fuzzy signature values for the structure in the example.

$$x = \begin{bmatrix} \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix} \\ \begin{bmatrix} 0.2 \\ \begin{bmatrix} 0.6 \\ 0.8 \\ 0.1 \end{bmatrix} \\ 0.9 \end{bmatrix} \\ \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix} \end{bmatrix}^T$$

After the aggregation operation is perform to the lowest branch of the structure, it will be described on higher level as:

$$x = \begin{bmatrix} 0.3 \\ \begin{bmatrix} 0.2 \\ 0.5 \\ 0.9 \end{bmatrix} \\ 0.1 \end{bmatrix}^T$$

Finally, the fuzzy signature structure will be:

$$x = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.1 \end{bmatrix}^T$$

Each of the signatures mentioned here contains information relevant to the particular data point x_0. By going higher in the signature structure, less information will be kept. In some operations it is necessary to reduce and aggregate information to become compatible with information obtained from another source (some detail variables missing or simply being locally omitted). In such cases interpolation within a fuzzy signature rule base is done, where the fuzzy signatures flanking an observation are not exactly of the same structure. In this case the maximal common sub-tree must be determined and all signatures must be reduced to that level in order to be able to interpolate between the corresponding branches or roots in some cases.

3 Example Application of Fuzzy Signature

Let S_{S_0} denote the set of all fuzzy signatures whose structure graphs are sub-trees of the structural ("stretching") tree of a given signature S_0. Then the signature sets introduced on S_{S_0} are defined by

$$A_{S_0} : X \to S_{S_0} .$$

In this case the prototype structure S_0 describes the "maximal" signature type that can be assumed by any element of X in the sense that any structural graph obtained by a set of repeated omissions of leaves from the original tree of S_0 might be the tree stretching the signature of some A_{S_0}.

$$\begin{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} \\ a_2 \\ \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} a_{411} \\ a_{412} \end{bmatrix} \\ a_{42} \end{bmatrix} \end{bmatrix}$$

An example for the usefulness of this definition is given below. Let us think about some patients, whose daily symptom signatures are based on doctors' assessments according to the following scheme:

$$A_S = \begin{bmatrix} \text{fever} \begin{bmatrix} \text{8 a.m.} \\ \text{12 p.m.} \\ \text{4 p.m.} \\ \text{8 p.m.} \end{bmatrix} \\ \text{blood pressure} \begin{bmatrix} \text{systolic} \\ \text{diastolic} \end{bmatrix} \\ \text{nausea} \\ \text{abdominal pain} \end{bmatrix} .$$

Let us take a few examples with linguistic values and numerical signatures:

$$A_1 = \begin{bmatrix} \begin{bmatrix} \text{none} \\ \text{none} \\ \text{slight} \\ \text{slight} \end{bmatrix} \\ \begin{bmatrix} \text{normal} \\ \varnothing \end{bmatrix} \\ \text{slight} \\ \text{slight} \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 0.2 \\ 0.2 \end{bmatrix} \\ \begin{bmatrix} 0.5 \\ \varnothing \end{bmatrix} \\ 0.25 \\ 0.25 \end{bmatrix} ,$$

$$A_2 = \begin{bmatrix} \begin{bmatrix} \varnothing \\ \varnothing \\ \text{moderate} \\ \text{moderate} \end{bmatrix} \\ \begin{bmatrix} \text{slightly high} \\ \text{rather high} \end{bmatrix} \\ \text{slight} \\ \text{none} \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} \varnothing \\ \varnothing \\ 0.4 \\ 0.4 \end{bmatrix} \\ \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix} \\ 0.25 \\ 0.0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \begin{bmatrix} \text{rather high} \\ \text{high} \\ \text{rather high} \\ \text{rather high} \end{bmatrix} \\ \begin{bmatrix} \text{rather high} \\ \text{very high} \end{bmatrix} \\ \text{none} \\ \varnothing \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} 0.8 \\ 0.6 \\ 0.8 \\ 0.8 \end{bmatrix} \\ \begin{bmatrix} 0.8 \\ 1.0 \end{bmatrix} \\ 0.0 \\ \varnothing \end{bmatrix}$$

(Ø stands for "not available"). Of course, normally the blood pressure values would initially rather be expressed by the physician as e.g. 120/75, which could then be converted to the linguistic values as appropriate for the patient, taking into account contextual information such as the higher normal blood pressure of infants and children and so on. As for most techniques, there is a significant role for the use of background knowledge of domain experts in data preprocessing.

Note that the structures above are different, which is an important point, namely, real world data is often like this, with missing components, or compressed parts. For patient 2, we have only 2 measurements for fever. The structure of the fuzzy signature contains some information by the association of vector components. The use of aggregation operators allows us to compare components irregardless of the different numbers of sub-components. Such aggregation operators would in general be designed for each vectorial component with the assistance of a domain expert. In this case, let us assume that the time of the day for fever is less significant, while the daily maximum value is most important. (By this assumption, the timing of temperature measurements must therefore be such that it ensure a reasonable coverage of the whole day.) The three signatures will be reduced to the following form, which is their maximal common sub-structure. (Note however, that in the case of the third signature, there is no data available on the presence or absence of abdominal pain, indicated by Ø in the original, nevertheless, this component was not eliminated from the other two, because of the high importance of this information in general.):

$$A_{1f} = \begin{bmatrix} 0.2 \\ \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \\ 0.25 \\ 0.25 \end{bmatrix}, A_{2f} = \begin{bmatrix} 0.4 \\ \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix} \\ 0.25 \\ 0.0 \end{bmatrix}, A_{3f} = \begin{bmatrix} 0.8 \\ \begin{bmatrix} 0.8 \\ 1.0 \end{bmatrix} \\ 0.0 \\ \varnothing \end{bmatrix}$$

The "fever component" can be verbally rewritten as "slight", "moderate" and "rather high", respectively. The signatures above still contain sufficient information about the "worst case fever" of each patient, while the detailed knowledge on the daily tendency of the fever is lost. This hierarchically structured access to the information is a key benefit of the fuzzy signatures.

We could continue this process further completely, and determine an overall "abnormal condition" measure $A_{1o} = [0.25]$, $A_{2o} = [0.4]$, $A_{3o} = [1.0]$. (Here the missing component being completely hidden already.)

4 Cognitive Modelling

Fuzzy signatures, as it has been described in the previous section, can address some issues of granulation and organisation well. In order to better model the human cognitive system, we have divided our cognitive modeling into two main categories. The first category consists of meta-levels of visual representation to model decision and cognitive behavior. For the ease of discussion, we will limit the model to a single meta-level in this paper. In this category the model consists of nodes and pointers to show the concepts and relations. Each node exhibits the behavior of a human cognitive system. Each node consists of three states, the sensory input state IN_i, current state CR_i, and action state AC_i. In the second category, nodes basically consist of the fuzzy signatures as described in the previous section. These signatures contain the knowledge necessary for the node to take any action.

Figure 2 shows a simple Fuzzy Cognitive Modeling. For node *i*,

$$N_i = (IN_i, CR_i, AC_i)$$

The modeling of the three states can be represented by the original definition of the fuzzy sets which is

$$A : X \rightarrow [0,1]$$

For some current states CR_i, if necessary, they will go down to the fuzzy signature level as

$$CR_i = A_{S_i}$$

where A_{s_i} is the fuzzy signature contributing to the knowledge of the node N_i.

There are basically two modes of operation for each fuzzy cognitive node: static and dynamic mode.

The static mode operation within the nodes is as follows:

- The three states within each node can be linked using fuzzy linguistics rules with antecedents and consequents.
- The antecedent of the fuzzy rules contains either the sensory input state, or the current state, or both the sensory input and the current state, i.e. (IN_i, CR_i).
- The consequent is the action state (AC_i).
- The operations between the antecedent/s and consequent are the ones used with fuzzy rules in general.
- Depending on how the fuzzy rules are constructed, the model may allow missing states within each node.
- When the action state is obtained, it can either propagate to the next node or the node can convert into dynamic mode.

Dynamic mode operations of the nodes are described by the following:

- In this mode, the time factor (*t*) is considered.
- Consequently, the fuzzy signature in the node will be formulated as $A_{s_i}(t)$.
- For (*t+1*), cross check with the fuzzy rules in the Fuzzy Cognitive Meta-level will be performed in order to see if the present node action can propagate to the next node. If not, the node will enter into (*t+2*). This is continued until an action can be propagated to the next connecting node/s, or when there exists a fuzzy rule to resolve the outcome.

For cases where there are more than one input arrows coming into the node, IN_i consists of more than one input component:

$$IN_i = \{IN_{i1}, IN_{i2}, ..., IN_{in}\}.$$

In order to avoid the combinatorial explosion problem within the rules, the relationship among $IN_{i1}, IN_{i2}, ..., IN_{in}$ is governed by a set of fuzzy aggregations, which eventually reduce the input state to a single component. The aggregations among them are not necessarily identical, even in the type. They can be a mixture of t-norms, s-norms, averaging aggregations and so on. Thus,

$$IN_i = IN_{i1} a_{1,2} IN_{i2} a_{2,3} ... a_{n-1,n} IN_{in}$$

Therefore, regardless of how many inputs are fed into the node, they will be resolved into a single fuzzy set before being used by the node. With this flexibility, missing information is allowed when performing modeling. For applications where computing power is crucial, or when the available information or data is massive, the nodes can be arranged in a distributed computing architecture, with each cognitive node being taken care of by separate nodes in the distributed computing cluster.

In those nodes where there is no input state, for example, node N_1 in Figure 2, the input state could be:

$$IN_1 = \varnothing$$

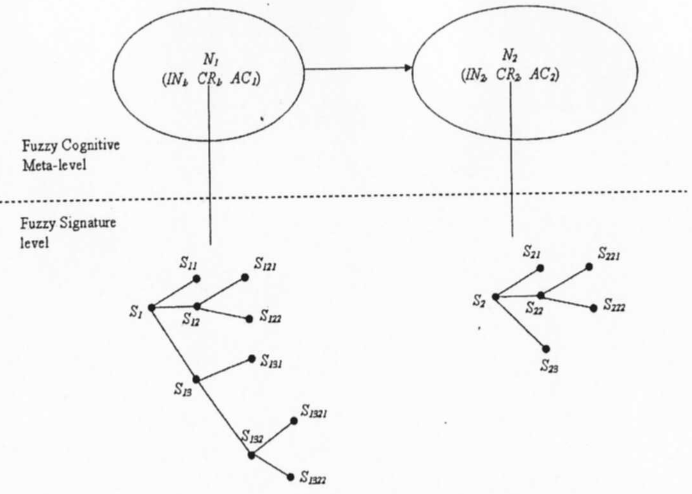

Fig. 2. The basic Efficient Fuzzy Cognitive Model

5 Conclusion

We described a technique for dealing with problems with complex and interdependent features or where data is missing. This was done by using the concept of fuzzy signatures, which extends the idea of vectorial fuzzy sets to components with varying numbers of sub-components. We also introduced a cognitive model to expand the usage of fuzzy signature for cases where complex decision is required. This hierarchical structuring allows the further use of domain experts as the information can be abstracted to higher levels, analogous to patterns of human expert decision making.

In the next phase, investigations will be done for the applicability of this new model to various problems where the complexity of the task, or its indeterministic and/or vague nature traditionally necessitate the involvement of human decision makers, such as in biology and medicine, agriculture, business, management and economics, and many others.

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